# UK Full Licence Maths

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# About the Author



My main passion is for engineering training and competence development. I was able to write a number of training manuals for instrumentation and control systems during my years with a major oil company and an extension to that, is this book, which I have really enjoyed writing.

On the radio front, I was first licensed as G6CVK in 1981 and as G4OKT the following year. During an assignment in the Sultanate of Oman I operated as A45WB. Now retired, my interests include 6 metres, CW, experimenting with antennas and a little DXing.

I taught the old RAE in the early 1990s and more recently have delivered Foundation/Intermediate/Advanced courses at Mold & District Radio Society. At the time of writing, I am an 'online' tutor for the Bath Based Advanced Distance Learning course.

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# Introduction

Many radio amateurs are successfully passing the UK Foundation and Intermediate examinations and are keen to take on the challenge of the Full course and examination.

There is quite a gap however, in terms of the competence required to handle the 'dreaded' mathematics. Many people have not studied maths for years and indeed they may not have even been taught the maths well, for many it remains one of those subjects that is just impossible to describe as FUN!

Having returned from overseas assignments, I have been teaching all three levels of licence and now I have some experience, I can see that maths is clearly the one subject that is preventing good radio amateurs from progressing further with the licence.

When attempting to understand the subjects of the advanced course you will see that the language changes a little, from English to maths, it is the language of electrical engineering and indeed physics in general, especially at this level, it is the only way that these engineering/physics concepts can be explained fully.

When you read maths books there is a great tendency for authors to jump steps, such that when a student is trying to follow the workings of an equation it is easy to become lost, especially as most students have not used maths for years. What I have attempted in this book, is to describe the maths in layman's terms and also include plenty of worked examples of every equation in the formula sheet that is issued for the UK advanced exam.

So good luck with your reading and please take your time to understand the examples.

# Preparing for the Full licence course

You will need a decent calculator, one with functions such as logarithms, squares, square roots, 1/x,  $(x^{-1})$  and so on. Included on each worked example within this book is a key press sequence to help you become familiar with your calculator.

Here are some typical functions you will use:

# Fig.1 Calculator functions



# **The Formula Sheet**

This is provided in the exam, it can also be found on the RSGB website as part of the Full Licence Exam Specification – (Section 2).

This book will provide examples of all the formulas required for the Full Licence exam, as listed in the exam specification formula sheet.

# **Units & Symbols**

In order to manipulate information for calculations we need to be aware of the symbols and units used, so here is a list used in the 'Full Licence' course for reference:

Physical entity	Symbol	Unit	Abbreviation	Typical use
Current	Ι	Ampere	А	μA, mA, A
Voltage	V	Volt	V	$\mu$ V, mV, V, kV,
				MV
Resistance	R	Ohm	Ω	Ω, k $Ω$ , M $Ω$
Capacitance	С	Farad	F	pF, nF, μF
Inductance	L	Henry	Н	μH, mH
Impedance	Ζ	Ohm	Ω	$\Omega, k\Omega, M\Omega$
Power	Р	Watt	W	μW, mW, W,
				kW, MW,
Energy	Е	Joule	J	μJ, mJ, J, kJ
Charge	Q	Coulomb	С	С
Frequency	f	Hertz	Hz	Hz, kHz, Mhz,
				Ghz
Period	Т	second	S	μs, ms, s
Wavelength	$\lambda_{(lambda)}$	metre	m	mm, m, km
Time constant	$\tau_{(tau)}$	second	S	μs, ms, s

# Table. 1 Units & Symbols

In the part of the table labelled 'Typical use', you can see that there are prefixes before the units such as p, n,  $\mu$ , m, k, M and G, these are used to represent very large or very small quantities of that particular entity such as power or volts.

In order to see how these prefixes are used we need to look at a subject called 'engineering notation'.

# Engineering notation

# **Dealing with Large units**

These are units such as kilohertz (kHz), Megahertz (Mhz), kilovolts (kV), Gigahertz (GHz).....

First of all let's start off with  $2x^2 = 4$ ....this is the same as  $2^2$ . Remember that any number multiplied by its self is 'squared', we have attached <sup>2</sup> to the back of the number to indicate that it is multiplied by itself.

Similarly we can see that  $2^3$  is the same as 2x2x2, which is equal to 8.  $2^4$  is the same as 2x2x2x2 = 16. In 'math-speak' this is known as raising the number 2 to the power of 2, 3 or 4 etc.

This applies to any number, but the one we are interested in is 10, this is the one used in engineering notation.

To represent large values we can utilize the process of raising to the power 2, 3 or 4 etc.

	Notation	Prefix	Meaning	
1	$10^{0}$		1	
10	10 <sup>1</sup>		10	
10x10	$10^{2}$			
10x10x10	$10^{3}$	Kilo (k)	1000	Thousand
10x10x10x10	10 <sup>4</sup>			
10x10x10x10x10	$10^{5}$			
10x10x10x10x10x10	$10^{6}$	Mega (M)	1000000	Million
10x10x10x10x10x10x10	$10^{7}$			
10x10x10x10x10x10x10x10x10	$10^{8}$			
10x10x10x10x10x10x10x10x10x10	$10^{9}$	Giga (G)	100000000	Billion

Table 2. Large number notation

As you can see from this, using a term such as  $10^6$  can save writing a lot of zeroes! As an example a frequency of 28 Megahertz, or 28 MHz as we would normally write, is expressed in engineering notation as 28 x  $10^6$ , meaning that it is 28 multiplied by 10, multiplied by itself 6 times, which is a million, we use the term Mega (with a capital M) to represent this.

When you use a calculator and have an answer in the display that is numerically very long, you can press the ENG key and reduce it to engineering units.

For example:

The frequency of 2945979.78 Hz, press ENG =  $2.94 \times 10^{6}$ Hz and because  $10^{6}$  means Mega our answer is 2.94MHz

The voltage of 1304.75 V, press ENG =  $1.304 \times 10^3$  V and because  $10^3$  = kilo, we have 1.304 kV

Notice that engineering units go in steps of 3, i.e.  $10^3$ ,  $10^6$ ,  $10^9$ 

# Dealing with small units

These are units such as millihenries (mH), microvolts (mV), nanofarads (nF), microamps (µA), picofarads (pF).

	Notation	Prefix	Meaning	
1/1	$10^{0}$		1	
1/10	10-1		0.1	Tenth
1/100	10-2		0.01	Hundredth
1/1000	10-3	milli (m)	0.001	Thousandth
1/10000	10-4			
1/100000	10-5			
1/1000000	10-6	micro (µ)	0.000001	Millionth
1/10000000	10-7			
1/10000000	10-8			
1/100000000	10-9	nano (n)	0.000000001	Billionth
1/10000000000	10-10			
1/10000000000	10-11			
1/100000000000	10 <sup>-12</sup>	Pico (p)	0.000000000001	

## Table 3. Small unit notation

Again the engineering units are in steps of 3, only this time to indicate they are being divided into 1 they have a minus sign i.e.  $10^{-3}$ ,  $10^{-6}$ ,  $10^{-9}$ .

If you have an answer in your calculator display that is a long number such as, 0.0000367, simply press the ENG key and you can display it as  $3.67 \times 10^{-5}$ , notice that this is not a familiar engineering unit so press ENG again and you will see  $36.7 \times 10^{-6}$ , which you may recognise as 'micro'.

You will have plenty of practice of engineering units during the course of this book.

Let's start by expressing some values as engineering units.

- Q1. Express 47microhenries (47µH) in engineering units
- Q2. Express 220picofarads (22pF) in engineering units
- Q3. What is  $14.22 \ge 10^{6}$  Hz
- Q4. What is  $3.3 \ge 10^3 \Omega$
- Q5. Express 456.78x10<sup>-7</sup> F in engineering units

Q1. We can see that the prefix is micro, which means multiply by  $10^{-6}$  So the key press sequence for the calculator would be:

47

 $x10^{x}$  (this button is on the bottom row of the calculator)

- (-) (on the Left side of the calculator)
- 6

You have expressed it correctly now, but if you now press = you will see it expressed as 47/1000000) Pressing ENG again gives you the original expression of  $47 \times 10^{-6}$ 

Q2. In a similar way, the prefix is pico which means multiply by  $10^{-12}$  The calculator key sequence is:

220 x10<sup>x</sup> (-) 12

pressing = gives you  $2.2 \times 10^{-10}$ pressing ENG again takes you back to  $220 \times 10^{-12}$  which is a more familiar way of expressing it.

- Q3. 14.22x10<sup>6</sup>Hz....referring to the table, 10<sup>6</sup> means Mega so the answer is **14.22 MHz**
- Q4.  $3.3 \times 10^3 \Omega$ ....here the  $10^3$  means kilo, so the answer is  $3.3 k\Omega$

Q5.  $456.78 \times 10^{-7}$  F, the key sequence is:

456.78 x10<sup>x</sup> (-) 7 = (gives  $4.5678 \times 10^{-5}$ ) ENG gives  $45.678 \times 10^{-6}$  (F) which is  $45.678 \mu$ F since  $\mu = 10^{-6}$ 

Notice that we have only moved the decimal point 1 place to the right. With practice you could do this part of the manipulation in your head without using the calculator.

# 20/08/2023 Calculating - total resistance of resistors in series

 $R_T = R_1 + R_2 + R_3 + \dots R_n$ 

 $R_T$  is the total resistance you want to calculate. The small *n* is used to denote that any number can be use, i.e. you can calculate for any number of resistors in series.

Notice that  $R_T$  is **the subject of the equation** and is always on the left hand side (LHS) of the 'equals' (=) sign.

This is simple addition.

Consider this example:

A circuit has 3 resistors R1=50 $\Omega$ , R2=100 $\Omega$ , R3=150 $\Omega$  connected in series. Calculate the total value of resistance in the circuit



# Fig.1 Resistors in series

Using our formula above:  $R_T = R1 + R2 + R3$   $R_T = 50 + 100 + 150$  $R_T = 300\Omega$ 

# Calculating - total resistance of resistors in parallel

1	_ 1	1	1	1
$\overline{R_T}$	$\overline{R_1}$	$\overline{R_2}$	$\overline{R_3}$	$\overline{R_n}$

 $R_T$  is again the total resistance you want to calculate, so as a final step in our calculation as you will see later, we have to invert our calculation using the  $\chi^{-1}$  button.

Q6.Three resistors R1=50 $\Omega$ , R2=100 $\Omega$ , R3=150 $\Omega$ , are connected in parallel, calculate the total resistance R<sub>T</sub>.



It is always a good idea to draw the circuit prior to the calculation, especially if it is not a simple one; this is what electrical engineers are taught to do early on in their studies.

## Fig.2 Resistors in parallel

Here we can see that with more than one path for the current to flow, the total resistance logically <u>must</u> be smaller than the smallest resistor, (since there are multiple paths for the current to flow) which in this case is R1 (50 $\Omega$ ).

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ $\frac{1}{R_2} = \frac{1}{50} + \frac{1}{100} + \frac{1}{150}$	The key sequence for this calculation is: 50 $x^{-1}$ + 100 $x^{-1}$
$\frac{1}{R_T} = \frac{11}{300}$ $R_T = \frac{300}{11} = 27.27\Omega$	+ 150 $x^{-1}$ = (shows answer of 11/300) $x^{-1}$ (shows Ans <sup>-1</sup> to indicate inversion) = (shows 300/11) S<>D 27.27 ( $\Omega$ )

Note: this value is less than the smallest resistor (50 $\Omega$ ).

NB. If you are not happy with having to press the SD button to get the answer to decimal rather than fraction, press SHIFT : MODE(setup) : 2 (Line10), the default will then be decimal.

Try another example with different resistor values

Q7. Three resistors are connected in parallel, R1=330 $\Omega$ , R2=1.2k $\Omega$ , R3=47k  $\Omega$ . Calculate the total resistance R<sub>T</sub>. In this example you can practice the use of engineering units - don't forget to <u>invert</u> the calculation result as a final step.

$$R_{1}$$

$$R_{2}$$

$$R_{3}$$

$$R_{7} = \frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}}$$

$$R_{7} = \frac{1}{3.885 \times 10^{-3}}$$

$$R_{7} = \frac{1}{3.885 \times 10^{-3}}$$

$$R_{7} = 257\Omega$$

# 20/08/2023 The 'product over sum' rule

This is used for two resistors (or inductors) in parallel only. (You can still use the traditional method of course).

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$
 Where the 'product' is the result of a multiplication X

.....and the 'sum' is the result of an addition +

Q8. What is the total resistance of a  $22\Omega$  resistor in parallel with a  $47\Omega$  resistor? (Just make R1=22 & R2 =47)



# Resistors of the same value

If you have 'n' parallel resistors, each with resistance R, the total resistance is given by  $R_T = R/n$ 

So- if you connect two  $100\Omega$  resistors in parallel, the resultant will be  $100/2 = 50\Omega$ . The same goes for three resistors in parallel, three  $100\Omega$  resistors in parallel will result in  $100/3 = 33.33\Omega$ . Four  $100\Omega$  resistors in parallel are  $100/4 = 25\Omega$ .

This can be put to good use when building QRP dummy loads since 3 resistors of  $150\Omega$  connected in parallel results in  $150/3 = 50\Omega$ .

# 20/08/2023 The parallel/series resistor network



# Fig.3 Parallel/Series network

A popular question in the intermediate exam involves calculating the total resistance of this network. First you need to calculate the value for the R1 & R2 parallel network and then ADD the result to the resistor R3 which is in series, (remember series resistances just add together).

To calculate the value of the R1 & R2 network, you can use the product over sum rule since it is the 'two' resistor case.

Q9. Using the circuit above,  $R1 = 220\Omega$ ,  $R2 = 180\Omega$  &  $R3 = 56\Omega$ . Calculate the total value of resistance  $R_T$  in the circuit.

First, calculate the value for the parallel network, we can call it  $R_{\rm P}$ .

$R_{P} = \frac{R_{1} \times R_{2}}{R_{1} + R_{2}}$ $R_{P} = \frac{220 \times 80}{220 + 80}$ $R_{P} = \frac{39600}{400}$ $R_{P} = 99\Omega$	Key sequence is: ( 220 X 180 ) ÷ ( 220 + 180 ) =99
--	--

Now we need to add this value to that of R3 which is in series So  $R_T = 99\Omega + 56\Omega = 155\Omega$ 

In practice, the exam question may include resistance values that have mixed units, so you may have to deal with kilo and Mega so I would practice using these.

Be careful not to rush your key presses on the calculator and don't be afraid to check you answers, there is plenty of time in the exam!

# 20/08/2023 **Ohm's Law**

This is a widely used law that you must be familiar with, if you are to have any chance of understanding electrical circuit functions.

The basic formula is:  $V = I \times R$  meaning that the voltage is simply the product of the current I multiplied by the resistance R.

This formula can be rearranged of course, some people like to use the 'triangle' method to remember how to make V, I or R, the subject of the equation:



If you wish to make the Current I, the subject of the equation, simply put your finger over the 'I' symbol and what you have left looks like V over R. If you need to make 'R' the subject, cover up R and you are left with V over I.

It is a useful reminder and the same idea can be used for  $P = I \times V$ , as we will see later.

BUT – you really need know how to rearrange formulas to make different components the subject without having to refer to triangles, this is called 'transposition' of the formula.

# Transposition of formulae

One of the basic rules of transposition is that whatever you do to the right hand side (RHS) of the equation, you must also do to the left hand side (LHS).

Let's take a closer look at Ohm's law.

 $V = I \times R$  .....if we need to find the current I, we need to get it by itself on one side of the equation, to do this we can divide both sides of the equation by R so we now have:

$$\frac{V}{R} = \frac{I \times R}{R}$$
 .....now these two Rs cancel each other out

which leaves us with....  $\frac{V}{R} = I$ 

Similarly if we want to make R the subject of the equation we need to get it by itself on one side. So we can divide both sides of the original equation by I:

.....now the Is cancel out

$$\frac{V}{I} = \frac{I \times R}{I} \quad \text{which leaves us with} \dots \quad \frac{V}{I} = R$$

This is the basis of most of the transposition you will be dealing with

Notice that if you move an expression from one side of the equation to the other it moves from the *numerator* to *denominator* and vice versa as a result of the move.

So if we look at the original  $V = I \times R$  expression, moving the R over to the LHS means it has to go in the *denominator* position, (beneath the V in this case).

Giving 
$$\frac{V}{R} = I$$

# Numerator and denominator

The number or expression above the fraction bar (the dividend) The number or expression below the fraction bar (the divisor) *Numerator Denominator* 

# **Power Current & Voltage**



Again, the relationship between the three variables can be expressed as a triangle. The method used to find the subject is the same as the Ohm's law triangle. It will come as no surprise then that the transposition method is the same. In the exam, either method can be used of course, it is up to you.

So,  $P = I \times V$  and if we need to find *I*, we need to get it by itself on one side of the equation, to do this we can divide both sides of the equation by V so we now have:

 $\frac{P}{V} = \frac{I \times V}{V} \quad \dots \dots \text{ now these two Vs cancel each other out}$ 

this leaves us with...  $\frac{P}{V} = I$ 

Similarly if we want to make V the subject of the equation we need to get it by itself on one side. So we can divide both sides of the original equation by *I*:

$$\frac{P}{I} = \frac{V \times V}{1} \dots \text{ now the Is on the RHS cancel out}$$

which leaves us with....  $V = \frac{P}{I}$ 

You can check this using the triangle method!

In the advanced licence course, the most difficult formula to transpose is the one for resonant frequency and once you can work through that, you will have all the transposition skills required for the advanced exam! This is dealt with in Appendix A.

Before moving on, let's attempt some (direct current related) Ohm's law and PIV questions.

Q10. A 12V car battery is used to supply a radio transceiver on the 2m band. The current drain is 10A, what is the total power consumed?

$P = I \times V$	Key sequence is:
$P = 10 \times 12$	10
P = 120 watts	X 12
1 = 120 Walls	=120

Q11. A torch bulb is supplied with 12V and draws 1.5A, what is it's resistance?

$R = \frac{V}{I}$	Key sequence is: 12
$R = \frac{12}{1.5}$ $R = 80$	÷ 1.5 = 8
N = 0.52	

Q12. A car headlamp is rated at 80W and is supplied with 12V, what current will flow through the lamp?

$I = \frac{P}{V}$	Key sequence is: 80	
$I = \frac{80}{12}$	÷ 12 = 6.66	
I = 6.66A		

You will notice that there are two other expressions we can use for power:

$$P = I^2 \times R$$
 and  $P = \frac{V^2}{R}$ 

You may be wondering where they came from - they are derived from

$$P = I \times V$$
 and  $V = I \times R$ 



Starting with  $P = I \times V$  .....we also know that  $V = I \times R$ 

So replacing V in the PIV formula with  $I \times R$ , gives  $P = I \times I \times R$ 

or simply  $P = I^2 \times R$ 

Similarly, if we change  $V = I \times R$  to  $I = \frac{V}{R}$  .....

we put this back into the  $P = I \times V$  equation.....

and we now have:  $P = V \times \frac{V}{R}$  or simply  $P = \frac{V^2}{R}$ 

We will use these variants of the power formula later.

In reality, the examiners for the advanced course will want to see an understanding of the use of engineering notation and you will not be required to derive these formulas in the exam. What is required is a question that includes both large and small numbers and also encompasses a few equations at once.

Let's try use of VIR and PIV with large and small numbers and then move on to a typical exam question.

Q13. A 12V car battery is used to supply a radio system. The current drain is 800mA, what is the total power consumed?

Key sequence is: 800 X10 <sup>x</sup> (-) 3 X 12
= 9.6

Q14. A small electronic circuit is supplied with 12V and draws 330µA, what is its resistance?

Key sequence is: 12
÷

$$R = \frac{V}{I}$$
$$R = \frac{12}{330 \times 10^{-6}}$$
$$R = 36 \times 10^{3}$$
$$R = 36k\Omega$$

Q15. The current in a circuit of total resistance  $47\Omega$  is 325mA, what is the power consumed?

Key sequence is: ( 325 X10 <sup>x</sup> (-) 3 ) x <sup>2</sup> x 47
47 = 4.96

Q16. A load resistor of value  $33k\Omega$  has a voltage across it of 6.3Volts. What is the power dissipated in the load?



P = 1.2 milliwatts

This next example is a more realistic standard for the exam, though it would probably occupy the space of three questions.



# Fig.4 Series/parallel network

Q17. What is the current flowing from the battery?

Q18. What is the voltage across R2?

Q19. What is the power dissipated in R3?

Q17. Answer

First, find the Total resistance of the circuit, (remember you did this earlier). In the exam the common configuration is one resistor in series connection with two in parallel. So as it is only two resistors in parallel, you can use the **'product over sum'** rule.

$R_{p} = \frac{R_{1} \times R_{2}}{R_{1} + R_{2}}$ $R_{p} = \frac{15 \times 10}{15 + 10}$ $R_{p} = \frac{150}{25}$ $R_{p} = 6\Omega$	Key sequence is: ( 15 X 10) ÷ ( 15 + 10 ) = 6
--	--

This 6 $\Omega$  can now be added to the series resistor R1 (20 $\Omega$ ) so the total circuit resistance, R<sub>T</sub> is 20+6 = 26 $\Omega$ 

This is a good start since we know the supply voltage is 12V and we know the resistance, we can calculate the current (I) flowing through the circuit using Ohm's law:

$I = \frac{V}{R_T}$	Key sequence is: 12
$I = \frac{12}{26}$	÷ 26 =0.46
I = 0.46A	

20/08/2023 Q18. Answer

The voltage across R2 is the same as R3 as they are connected in parallel, (which is the same as an equivalent  $6\Omega$  resistor).

The current flowing in the whole circuit is 0.46A and this also flows through the parallel combination of R2 & R3. We can find the voltage across this combination using Ohm's law.

$V = I \times R$	Key sequence is:
$V = 0.46 \times 6$	0.46
$V = 0.40 \times 0$	Х
V = 2.76Volts	6
	=2.76

#### Q19. Answer

The power dissipated in R3 is given by P=IV, but we do not have a value for I within the parallel combination, so we can use:



# Potential divider calculation





The supply voltage  $V_{in}$  will cause a current to flow in the two series connected resistors, R1 & R2.

where (using Ohms law) the total current  $I = \frac{V_{in}}{R_1 + R_2}$ 

The voltage across  $R_2$  (we can call  $V_{out}$ ) is equal to  $I \times R_2$ 

(again just using Ohm's law)



....remember, this part is equal to I

Similarly, if you want to make the voltage  $V_{out}$  to be across  $R_1$ , the equation becomes:

$$V_{out} = \frac{V_{in}}{R_1 + R_2} \times R_1$$

# **Calculations with Capacitors**

The value of a capacitor determines how much charge it will store for a given potential difference.

 $C(Farads) = \frac{Q(coulombs)}{V(volts)}$ 

# Capacitors with the same value

If you have 'n' capacitors in series, each with capacitance C, the total capacitance is given by  $C_T=C/n$ 

# Capacitors in series

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

# Two capacitors in series 'product over sum' rule

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2}$$

# Capacitors in parallel

 $C_T = C_1 + C_2 + C_3 + \dots + C_n$ 

# Relationship between area, distance between plates and



Q20. A capacitor is formed by two parallel plates. What should be done to make the capacitance four times greater?

- A) Double the area of the plates and double the distance between them
- B) Double the area of the plates and halve the distance between them
- C) Halve the area of the plates and halve the distance between them
- D) Halve the area of the plates and double the distance between them

Answer:

- 'A' is wrong since you are not changing the ratio
- 'B' looks good since the ratio is now 4:1
- 'C' is wrong as the ratio has not changed
- 'D' is wrong, the capacitor will become smaller
- Q21. What is the total capacitance of the two capacitors C1=60pF and C2=120pF, connected in series as below?

C1 C2 . .∎ (∎ (■		
	Fig.6 Capacitors in series	Key sequence is: 60
$\frac{1}{C_T} = \frac{1}{60 \times 10^{-12}} + \frac{1}{120 \times 10^{-12}}$		X10 <sup>×</sup> (-) 12
$\frac{1}{C_T} = 1.66 \times 10^{10} + 8.33 \times 10^9$		X <sup>-1</sup> + 120
$\frac{1}{C_T} = 2.5 \times 10^{10}$		X10 <sup>×</sup> (-) 12
$C_T = \frac{1}{2.5 \times 10^{10}}$		$X^{-1}$ =2.5x10 <sup>10</sup> $X^{-1}$
$C_T = 40 \times 10^{-12} (40  pF)$		= 4x10 <sup>-11</sup>

NB For two capacitors in series you can use the product over sum rule!

Q21 answer again!

$$C_{T} = \frac{C_{1} \times C_{2}}{C_{1} + C_{2}}$$
$$C_{T} = \frac{60 \times 120}{60 + 120}$$
$$C_{T} = \frac{7200}{180}$$
$$C_{T} = 40(pF)$$

There is no need to add the engineering notation in the calculations as both capacitors are in pF, so be aware of these simple approaches.

**Q22.** What is the total capacitance of C1=60pF and C2=120pF in parallel? C1  $\sim$  C1



# Fig.7 Capacitors in parallel

$$C_{T} = C_{1} + C_{2}$$

$$C_{T} = 60 \times 10^{-12} + 120 \times 10^{-12}$$

$$C_{T} = 180 \times 10^{-12}$$

$$C_{T} = 180 pF$$

Key sequence is:  

$$60$$
  
 $X10^{x}$   
(-)  
 $12$   
+  
 $120$   
 $X10^{x}$   
(-)  
 $12$   
 $=1.8x10^{-10}$   
ENG gives  $180x10^{-12}$   
( $180pF$ )

Q23. In the circuit below, C1 =120pF, C2=20pF and C3=20-120pF. Over what range will the total capacitance vary?



The 'product over sum' rule is very effective here. First take a look at the parallel section. The starting point will be two 20pF capacitors in parallel, so these simply add to form a 40pF capacitor, so now we have a 120pF and a 40pF in series. Now use the product over sum rule to calculate the total value.

$$C_{T} = \frac{C_{1} \times C_{2}}{C_{1} + C_{2}}$$
$$C_{T} = \frac{120 \times 40}{120 + 40}$$
$$C_{T} = \frac{4800}{160}$$
$$C_{T} = 30(pF)$$

At the other end of the tuning range we have a parallel combination of 20pF

in parallel with 120pF, which results in a total of 140pF. So the total is 140pF in parallel with 120pF. Again use the 'product over sum' rule to calculate the total capacitance.

$$C_{T} = \frac{C_{1} \times C_{2}}{C_{1} + C_{2}}$$
$$C_{T} = \frac{120 \times 140}{120 + 140}$$
$$C_{T} = \frac{16800}{260}$$
$$C_{T} = 64(pF)$$

So the total range is 30-64pF.

# Charging & discharging a capacitor through a resistor

The capacitor is considered fully charged after 5 time constants, or  $5\tau$ 



Fig.8 Capacitor charge & discharge



# Fig.9 Capacitor charge & discharge curves

Time T=0 (at the bottom left corner of the graph), represents the time of the closure of switch S1, which allows current to flow through the resistor to the capacitor. After one time constant ( $\tau$ ), the voltage across the capacitor is 63.2% of maximum and after 5 time constants, the capacitor is fully charged.

Charging a capacitor through a resistor  $\tau = C \times R$ 

Where au is the time constant (in seconds)

Q24. What is the time constant if the resistor is  $47\mu$ F and the 'limiting' resistor is  $33k\Omega$ ?

Key sequence is: 47 X10 <sup>×</sup> (-) 6 x 47 X10 <sup>×</sup> 3 =1.55s

Q25. How long would it take to fully charge the capacitor in Q24?

Answer:  $5 \times \tau = 5 \times 1.55 = 7.75s$ 

Q26. A 22 $\mu$ F capacitor is being charged via a 47 $\Omega$  resistor. The supply voltage is 24 volts, how long would it take for the voltage across the capacitor to reach 15volts?

#### Answer:

15 volts is 63% of 24, (15/24 = 0.63) so this is equal to 1 time constant. So, we need to calculate the time constant and that is our answer!

$\tau = 22 \times 10^{-6} \times 47$ $\tau = 1.034 \times 10^{-3}$ $\tau = 1ms$	Key sequence is: 22 X10 <sup>x</sup> (-) 6 x 47 =1.034x10 <sup>-3</sup>
	Approx 1ms

# Resistor & capacitor in series

Adding voltages in a series circuit requires vector addition, since the voltages across R and C are 90° from each other.



When the circuit is supplied with an a.c. supply the voltage across the resistor is in phase with the current and it can be calculated simply, using Ohms Law. The voltage across the capacitor lags the current by 90° and is calculated using V=I x  $X_C$ 

The current I is common to both R and C and is treated as the reference vector. The voltage across the resistor is in phase with the current, so the voltage vector is drawn parallel to the current vector.

The voltage across the capacitor lags the current by  $90^\circ$  and is drawn downwards. The vector addition of  $V_{\rm C}$  and  $V_{\rm r}$  is equal to the supply voltage.

# Fig.10 Vector addition of $V_{R}$ and $V_{C}$

Using the Pythagoras theorem we can conclude that:

 $V_{Supply} = \sqrt{V_R^2 + V_C^2}$ 

Given any two parameters we can then re-arrange the formula to find the missing one.

Q27. In a series RC circuit with a supply voltage of 10Vrms, if the voltage across R=6.855V what is the voltage across the capacitor C?

$$V_{S}=10V_{rms}$$

$$V_{S}=V_{R}^{2}+V_{C}^{2}$$

$$V_{S}=V_{R}^{2}+V_{C}^{2}$$

$$V_{S}=V_{R}^{2}+V_{C}^{2}$$

$$V_{C}=\sqrt{V_{S}^{2}-V_{R}^{2}}$$

$$V_{C}=\sqrt{V_{S}^{2}-V_{R}^{2}}$$

$$V_{C}=\sqrt{10^{2}-6.855^{2}}$$

$$V_{C}=\sqrt{100-47}$$

$$V_{C}=\sqrt{53}$$

$$V_{C}=7.28Vrms$$



# Resistor & inductor in series

# Fig.11 Vector addition of $V_{R}$ and $V_{L}$

The voltage across an inductor leads the current by 90°, this is shown in Fig 11.

In this case  $V_{Supply} = \sqrt{V_R^2 + V_L^2}$  which is similar in approach to the series RC circuit.

Q28. In a series RL circuit with a supply voltage of 20Vrms, if the voltage across the resistor R is 8.75V, what is the voltage across the inductor?



 $L_T = L_1 + L_2 + L_3 + \dots + L_n$ 

# Inductors in parallel

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_n}$$

For two inductors in parallel you can also use the product over sum rule.

$$L_T = \frac{L_1 \times L_2}{L_1 + L_2}$$

Time constant of circuit containing inductance and resistance



Fig.12 Inductor charge circuit



Fig.13 Inductor charge curve

Q29. A circuit consisting of an inductor and resistor in series has a time constant of 15ms (milliseconds). After voltage is applied how long will it take to achieve the final value of current?

Answer: 5 time constants =  $5 \times 15ms = 75ms$ .

# Alternating currents and voltages



### Fig.14 The sinusoidal waveform (sine wave)

90° represents 1/4 cycle 180° represents 1/2 cycle 270° represents 3/4 cycle 360° represents 1 full cycle

The frequency f is the number of complete cycles per second, where 1 cycle per second = 1Hertz (Hz).

The period T is the time taken to complete one cycle.

These two parameters are linked by the formulas: T=1/f and f=1/T

# The peak value is not the one that you measure with your digital multi-meter; the meter measures the RMS or root mean square value.

This RMS value of the voltage of the sine wave is equivalent to the DC voltage that would have the same heating effect, or the same overall rate of energy conversion, or work done.

Again, there is a relationship between RMS and peak voltage:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$
 or  $0.707 \times V_{peak}$ 

There is usually an exam question that will ask you to convert from peak to RMS values.

Q30.An alternating current in the form of a sine wave has a peak value of 24V, what is its RMS value?

$$V_{rms} = 0.707 \times V_{peak}$$
$$V_{rms} = 0.707 \times 24$$
$$V_{rms} = 17 volts$$

#### Q31. If an RMS voltage has a value of 15volts, what is its peak value?

Now we must transpose the formula for the original formula RMS value, to make 'Vpeak' the subject.

$$V_{rms} = 0.707 \times V_{peak}$$
  
Divide both sides of the equation by 0.707  
$$\frac{V_{rms}}{0.707} = \frac{0.707 \times V_{peak}}{0.707}$$
  
These two then cancel out  
These two then cancel out

So for an RMS value of 15volts 
$$V_{peak} = \frac{V_{rms}}{0.707}$$
  
 $V_{peak} = \frac{15}{0.707}$   
 $V_{peak} = 21.2volts$ 

Q32. If the time for one complete cycle is 0.0002 ms (milliseconds), what is the frequency?

 $T = \frac{1}{f}$  Q33. What is the time period of a 3.5MHz signal?  $T = \frac{1}{3.5 \times 10^{6}}$   $T = 285 \times 10^{-9}$ Key sequence is: 3.5 X10<sup>x</sup> 6 x<sup>-1</sup> Q34. What is the wavelength  $\lambda$  of an RF transmission of frequency 52 MHz?

The relationship between frequency and wavelength is given by  $\lambda = v/f$ , where v = the velocity of light in free space (vacuum). Generally quoted as c (special case) =  $3 \times 10^8$  m/s.

now,  $c = 3x10^8$  m/s. is the same as  $c = 300x10^6$  m/s so if we now change the equation to:  $\lambda = 300/f$ , where the frequency is in MHz, this makes the conversion easier since the  $x10^6$  parts of v and f now cancel out!

$$\lambda = \frac{v}{f}$$
$$\lambda = \frac{300 \times 10^6}{52 \times 10^6}$$
$$\lambda = \frac{300}{52}$$
$$\lambda = 5.77m$$

# Capacitive reactance

$$X_c = \frac{1}{2\pi fC}$$

Where f is frequency in Hz and C is capacitance in Farads

Q35. A  $5\mu$ F capacitor is connected across the 240V 50Hz mains supply. What is the reactance at that frequency and how much current will flow?

$$\begin{split} X_{c} &= \frac{1}{2\pi fC} \\ X_{c} &= \frac{1}{2\times 3.14 \times 50 \times 5 \times 10^{-6}} \\ X_{c} &= \frac{1}{1.57 \times 10^{-3}} \\ X_{c} &= 637\Omega \end{split} \qquad \begin{array}{c} \text{Key sequence is:} \\ 2 \\ 3.14 \\ \times \\ 50 \\ 5 \\ \times \\ \times \\ 10^{\times} \\ (-) \\ 6 \\ X^{-1} \\ = 637\Omega \end{split}$$

We can now use Ohm's law to calculate the current, where  $I = \frac{V}{V}$ 

$$I = \frac{X_c}{X_c}$$
$$I = \frac{240}{637}$$
$$I = 0.377 A(RMS)$$

Q36. At what frequency would a 22nF capacitor have a reactance of  $50\Omega$ ?

$$X_{C} = \frac{1}{2\pi fC}$$

$$fX_{C} = \frac{1}{2\pi fC} \times f$$
Multiply both sides by f
$$fX_{C} = \frac{1}{2\pi C}$$

$$\frac{fX_{C}}{X_{C}} = \frac{1}{2\pi CX_{C}}$$
Divide both sides by X<sub>C</sub>

$$f = \frac{1}{2\pi CX_{C}}$$

To simplify the process, if it is on the numerator on the LHS it goes to the denominator on the RHS, so simply swap places with the f and  $X_C$ .



Q37. What capacitance would be required to give a reactance of  $750\Omega$  at a frequency of 50kHz?

$X_C = \frac{1}{2\pi fC}$	Key sequence is: 2
$C = \frac{1}{2\pi f X_C}$	X 3.14 X
$C = \frac{1}{2 \times 3.14 \times 50 \times 10^3 \times 750}$	50 X10 <sup>x</sup> 3 X
$C = 4.25 \times 10^{-5}$ $C = 4.25 nF$	750 $X^{-1}$ =4.25x10 <sup>-9</sup> =4.25nF

NB. C and  $X_C$  swap places this time.

# Inductive reactance

 $X_l = 2\pi f L$ 

Q38. An inductor of value 22mH is operating at a frequency of 28.5MHz, what is it's reactance?

Q39. At what frequency would a 22mH inductor have a reactance of  $500\Omega$ ?

$X_{L} = 2\pi fL$ $f = \frac{X_{L}}{2\pi L}$ $f = \frac{500}{2 \times 3.14 \times 22 \times 10^{-3}}$ $f = 3.618 \times 10^{3}$ f = 3.618 kHz	Key sequence is: 500 ÷ ( 2 X 3.14 X 22 X10 <sup>×</sup> (-) 3
<i>j</i> = 5.010 <i>k</i> 112	(-) 3 ) = $3.618 \times 10^3$ = $3.618$ kHz

Q40. What inductance would be required to give a reactance of  $750\Omega$  at a frequency of 50kHz?

$Y = 2\pi f I$	Key sequence is:
$L = \frac{X_L}{2\pi f}$	; ; ( 2
$L = \frac{750}{2 \times 3.14 \times 50 \times 10^3}$ L = 2.39mH	X 3.14 X 50
L – 2.57mm	X10 <sup>x</sup> 3 ) =2.39x10 <sup>-3</sup>
	=2.39mH

# Impedance

$$Z = \sqrt{R^2 + X^2}$$

Q41. A resistor of  $200\Omega$  and a capacitor of 47nF are fed from an AC source. At a certain frequency the reactance of the capacitor is  $20\Omega$ , what is the impedance of the circuit at that frequency?

$Z = \sqrt{R^2 + X^2}$	Key sequence is: 200
$Z = \sqrt{200^2 + 20^2}$	x <sup>2</sup> +
$Z = \sqrt{40000 + 400}$	20 x <sup>2</sup>
$Z = \sqrt{40400}$	=40400 √
$Z = 201\Omega$	=201

Q42. A resistor of  $470\Omega$  and a capacitor of 180pF are fed from an a.c. source of 14MHz.What is the impedance of the circuit at that frequency?

We first need to find the capacitive reactance  $X_{C}$ 

$$X_{c} = \frac{1}{2\pi fC}$$

$$X_{c} = \frac{1}{2 \times 3.14 \times 14 \times 10^{6} \times 180 \times 10^{-12}}$$

$$X_{c} = \frac{1}{15.82 \times 10^{-3}}$$

$$X_{c} = 63\Omega$$

Now we can put this into the formula for impedance Z:

$$Z = \sqrt{R^2 + X_c^2}$$
$$Z = \sqrt{470^2 + 63^2}$$
$$Z = \sqrt{220900 + 3969}$$
$$Z = 474\Omega$$

Q43. A resistor of  $470\Omega$  and a inductor of  $180\mu$ H are fed from an AC source of 14MHz. What is the impedance of the circuit at that frequency?

We first need to find the inductive reactance  $\boldsymbol{X}_{\!L}$ 

$$X_{L} = 2\pi fL$$
  

$$X_{L} = 2 \times 3.14 \times 14 \times 10^{6} \times 180 \times 10^{-6}$$
  

$$X_{L} = 15.8 \times 10^{3}$$
  

$$X_{L} = 15.8 k\Omega$$

Now we can put this into the formula for impedance Z:

$$Z = \sqrt{R^2 + X_L^2}$$
  

$$Z = \sqrt{470^2 + (15.8 \times 10^3)^2}$$
  

$$Z = \sqrt{220900 + 249 \times 10^6}$$
  

$$Z = \sqrt{249.2 \times 10^6}$$
  

$$Z = 15.78 \times 10^3$$
  

$$Z = 15.78 k\Omega$$

This value is very close to that of  $X_{C_{s}}$  which is no surprise, because it is so large when added as a vector with R (which is small).

# **Resonant Frequency**

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Q44. A circuit consists of an inductance of 33mH in parallel with a 50pF capacitor, what is the resulting resonant frequency?

$$\begin{split} f_r &= \frac{1}{2\pi\sqrt{LC}} & \text{Key sequence is:} \\ 2 \\ \chi \\ f_r &= \frac{1}{2\times 3.14\sqrt{33\times 10^{-3}\times 50\times 10^{-12}}} & \text{Xey sequence is:} \\ 2 \\ \chi \\ 3.14 \\ \chi \\ \sqrt{33} \\ 10^{x} \\ (-) \\ 3 \\ \chi \\ 50 \\ \chi \\ 10^{x} \\ (-) \\ 3 \\ \chi \\ 50 \\ \chi \\ 10^{x} \\ (-) \\ 12 \\ f_r &= \frac{1}{8.1\times 10^{-6}} & (-) \\ 12 \\ f_r &= 123\times 10^{3} \\ f_r &= 123kHz & \text{End } 123456 \\ \text{ENG gives } 123456 \\ \text{ENG gives } 123.45 \\ \chi \\ (123kHz) \\ \end{split}$$

Q45. At 14MHz an antenna presents an inductive reactance at its feed-point equivalent to an inductor of 42µH. What value of capacitance is required to make the antenna resonant?

First we need to use the above formula but transposed to make C the Subject:

$$C = \frac{1}{4\pi^2 f_r^2 L}$$

NB An explanation of this transposition is in Appendix A.

$C = \frac{1}{4\pi^2 f_r^2 L}$ Answer:	Key sequence is: 4 X
$C = \frac{1}{4 \times 3.14^2 \times (14 \times 10^6)^2 \times 42 \times 10^{-6}}$	3.14 X <sup>2</sup> (
$C = \frac{1}{39.438 \times 1.96 \times 10^{14} \times 42 \times 10^{-6}}$	14 X10 <sup>x</sup> 6
$C = \frac{1}{324 \times 10^9}$	) X <sup>2</sup> X 42
$C = 3.08 \times 10^{-12}$ $C = 3 pF$	X10 <sup>×</sup> (-) 6
	= gives 3.24 X10 <sup>11</sup> X <sup>-1</sup> gives 3.08 X10 <sup>-12</sup> (3pF)

Q46. At 3.5MHz an antenna presents a capacitive reactance at its feed-point equivalent to a capacitor of 47pF. What value of inductance is required to make the antenna resonant?

(44µH)

1	
$L = \frac{1}{4\pi^2 f_r^2 C}$	Key sequence is: 4
$L = \frac{1}{4 \times 3.14^2 \times (3.5 \times 10^6)^2 \times 47 \times 10^{-12}}$	X 3.14 X <sup>2</sup>
$L = \frac{1}{39.438 \times 1.225 \times 10^{13} \times 47 \times 10^{-12}}$	( 3.5
$L = \frac{1}{22.7 \times 10^3}$	6 )
$L = 44 \times 10^{-6}$ $L = 44 \mu H$	X <sup>2</sup> X 47
	X10 <sup>x</sup> (-)
	12 = gives 22706 X <sup>-1</sup> gives 4.4 X10 <sup>-5</sup> ENG gives 4.4 X10 <sup>-6</sup>

# Series RLC Circuit example



An RLC circuit where R=10k $\Omega$ , L=246mH and C=5nF

Q47. What is the total impedance of the circuit at resonance?

At resonance, the reactive components  $X_C \& X_L$  cancel out to leave the resistive component R only so the impedance will be  $10k\Omega$ .

Q48. If the source voltage is 36V what will be the voltage across R, L and C at resonance?

At resonance, the voltage across R is the applied voltage V. Current through the resistor is  $I_R = V/R$ . This current flows through both the capacitor and the inductor.

So the voltage across the capacitor 
$$V_C = \frac{V}{R} \times \sqrt{\frac{L}{C}}$$
  
 $V_C = \frac{36}{10 \times 10^3} \times \sqrt{\frac{246 \times 10^{-3}}{5 \times 10^{-9}}}$   
 $V_C = 3.6 \times 10^{-3} \times 7014$   
 $V_C = 25.25V$ 

This same voltage is across the inductor but 180° out of phase with the voltage across the capacitor so cancels out.

Q49. What is the resonant frequency of the circuit?

$f_r = \frac{1}{2\pi \sqrt{IC}}$	Key sequence is: 2 x	
$f_r = \frac{1}{2 \times 3.14 \times \sqrt{246 \times 10^{-3} \times 5 \times 10^{-9}}}$	^ 3.14 X √	
$f_r = \frac{1}{6.28 \times \sqrt{1.23 \times 10^{-9}}}$	246 X10 <sup>x</sup> (-) 3	
$f_r = \frac{1}{6.28 \times \sqrt{1.23 \times 10^{-9}}}$	X 5 X10 <sup>x</sup>	
$f_r = 4540Hz$ $f_r = 4.54kHz$	(-) 9 = (2.2x10 <sup>-4</sup> ) $y^{-1}$	
	^ = 4540 (4.54kHz)	

Q50. What is the Q factor of the circuit?

$$Q = \frac{2\pi fL}{R}$$

$$Q = \frac{2 \times 3.14 \times 4540 \times 246 \times 10^{-3}}{10 \times 10^{3}}$$

$$Q = \frac{7014}{10 \times 10^{3}}$$

$$Q = 0.7$$

# Dynamic impedance/resistance

For a parallel LC circuit at resonance, the reactive components cancel and we are left with a resistive component  $R_D$  only, where  $R_D = L/CR$ 

Q51. What is the impedance at resonance of a parallel tuned circuit comprising a capacitor of 47pF and a coil of  $22\mu$ H with an internal resistance of  $12\Omega$ ?



Fig.13 Parallel tuned circuit

$R_D = \frac{L}{CR}$ $R_D = \frac{22 \times 10^{-6}}{(47 \times 10^{-12} \times 12)}$	Key sequence is: 22 X10 <sup>×</sup> (-) 6
$R_{D} = \frac{22 \times 10^{-6}}{5.64 \times 10^{-10}}$ $R_{D} = 39 \times 10^{3}$ $R_{D} = 39k\Omega$	$\div$ (47 X10 <sup>x</sup> (-) 12 X 12) = 39007 ENG 39 X 10 <sup>3</sup>

# **Transformer Ratios**

$$V_{s} = V_{p} \frac{N_{s}}{N_{p}}$$
$$I_{p} = I_{s} \frac{N_{s}}{N_{p}}$$
$$Z_{p} = Z_{s} \left(\frac{N_{p}}{N_{s}}\right)^{2}$$

Q52. A transformer has 200 turns on the secondary coil and 600 turns on the primary coil. A voltage of 300VAC is applied across the primary coil, what is the output voltage across the secondary coil?



# Fig.14 Transformer ratios

$V_{S} = V_{P} \times \frac{N_{S}}{N_{P}}$	Key sequence is: 300
$V_s = 300 \times \frac{200}{600}$	X ( 200
$V_s = 300 \times \frac{1}{3}$	÷ 600 )
$V_s = 100 volts$	=100

Q53. A transformer has 500 turns on the secondary coil and 200 turns on the primary coil. The current in the secondary coil,  $I_s = 2A$ , what is the value of the primary coil current  $I_p$ ?



Q54. An amplifier requires a load impedance of 2.45k **Ω**, which needs to be matched to provide a final output impedance of 50 **Ω**. If the secondary has 6 turns, how many turns should there be on the **primary** to give Z<sub>p</sub>= 2.45kohms and Z<sub>s</sub>=50 ohms?

Here, we need to transpose for N<sub>p</sub> (all the steps are left in for clarity)



$$\begin{split} & Z_{P} = Z_{S} \left( \frac{N_{P}}{N_{S}} \right)^{2} \\ & \frac{Z_{P}}{Z_{S}} = \frac{Z_{S} \left( \frac{N_{P}}{N_{S}} \right)^{2}}{Z_{S}} \\ & \text{Divide both sides by } Z_{S} \\ & \text{The } Z_{S} \text{ s cancel out to leave} \\ & \frac{Z_{P}}{Z_{S}} = \left( \frac{N_{P}}{N_{S}} \right)^{2} \\ & \sqrt{\frac{Z_{P}}{Z_{S}}} = \sqrt{\left( \frac{N_{P}}{N_{S}} \right)^{2}} \\ & \sqrt{\frac{Z_{P}}{Z_{S}}} = \frac{N_{P}}{N_{S}} \\ & \sqrt{\frac{Z_{P}}{Z_{S}}} \times N_{S} = \frac{N_{P}}{N_{S}} \\ & \sqrt{\frac{Z_{P}}{Z_{S}}} \times N_{S} = \frac{N_{P}}{N_{S}} \\ & \sqrt{\frac{Z_{P}}{Z_{S}}} \times N_{S} = N_{P} \\ & \sqrt{\frac{Z_{P}}{Z_{S}}} \times N_{S} = N_{P} \\ & \sqrt{\frac{2450}{50}} \times 6 = 42turns \end{split}$$

# The Decibel

The decibel is widely used in RF engineering as it makes relative measurements easy. Specifically, the decibel is a ratio. For Foundation and Intermediate courses, the examples used simply necessitate the use of the '3 & 10' rule only. For example, every time you double the power, you are increasing the power by 3dB and every time you multiply the power by 10, you are increasing by 10dB, also, every -3dB halves the figure and -10dB is a tenth, from this, most simple examples can be determined by mental arithmetic.

As a reminder:

Gain (dB)	Power multiplication	
3	Two times	2X
6	Four times	2X2=4
9	Eight times	2X2X2=8
10	Ten times	10X
16 (10+3+3)	Forty times	10X2X2 = 40
7 (10-3)	Five times	10X0.5=5

The basic formula is: 
$$Gain(loss) = 10 \log_{10} \left( \frac{Power_{out}}{Power_{in}} \right) dB$$

So, in practice a negative number such -5.6dB is considered a loss and a positive number such as 4dB is considered a gain, (we do not normally use +4dB, the + sign is omitted and the number is assumed to be positive).

For the Advanced exam however, you will be required to use the formula so in the following pages you will see a few examples.

# Effective Radiated Power – ERP

ERP =Power X Gain (linear)

Q55. What is the gain of an antenna (in decibels), of the effective radiated power is 50W when an input of 25W is applied?

$$Gain = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$$
  

$$Gain = 10 \log_{10} \left( \frac{50}{25} \right)$$
  

$$Gain = 10 \log_{10} 2$$
  

$$Gain = 10 \times 0.3$$
  

$$Gain = 3dB$$
  
Key sequence is:  

$$\log_{50} 2$$
  

$$= 0.3$$
  

$$\times 10$$
  

$$= 3$$

This represents a gain of X2 i.e. 50watts effective power output, for an input of 25watts.

Q56. An antenna for 2m has a gain of 8dB and is fed with an RF power of 25W, what is the erp?

We first need to find the linear gain equivalent to 8dB, this is simply the ratio of power out  $\div$  power in.

$$Gain(dB) = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$$
  

$$\frac{P_{out}}{P_{in}} = A \log_{10} \left( \frac{gain(dB)}{10} \right)$$
  

$$\frac{P_{out}}{P_{in}} = A \log_{10} \left( \frac{8}{10} \right)$$
  

$$\frac{P_{out}}{P_{in}} = 6.3$$
  
Key sequence is:  
Shift log  
8  
÷  
10  
=6.3

Now we can substitute this back in to the erp formula:

 $erp = power \times gain$  $erp = 25 \times 6.3$ erp = 157.5watts

# Calculating field strength

$$E = \frac{7\sqrt{erp}}{d}$$

Q57. What is the field strength at a distance of 20m from an antenna with an erp of 25 Watts?

$E = \frac{7\sqrt{erp}}{d}$	Key sequence is: 7
$E = \frac{7\sqrt{25}}{20}$	x √ 25 −35
$E = \frac{35}{20}$	÷ 20
E = 1.75 v / m	=1.75

Q58. A 50 watt transmitter is feeding an antenna with a gain of 8dB via a coaxial feeder with a loss of 2dB. What is the field strength 30m from the antenna?

The e.r.p. from the antenna is  $(8dB - 2dB) \ge 50$  Watts

erp =6dB x 50 watts \*Remember that each 3dB is a doubling of the power, so:

 $erp = 2 \ge 2 \ge 50 = 200$  Watts

$$E = \frac{7\sqrt{erp}}{d}$$
$$E = \frac{7\sqrt{200}}{30}$$
$$E = \frac{7\times14.14}{30}$$
$$E = 3.3V / m$$

Q59. An antenna has an erp of 100 Watts, at what distance from the antenna would the field strength be 5V/m?

$$E = \frac{7\sqrt{erp}}{d}$$
$$d \times E = \frac{7\sqrt{erp}}{A} \times d$$
$$d \times E = 7\sqrt{erp}$$
$$\frac{d \times E}{E} = \frac{7\sqrt{erp}}{E}$$
$$d = \frac{7\sqrt{erp}}{E}$$
$$d = \frac{7\sqrt{erp}}{5}$$
$$d = \frac{70}{5}$$
$$d = 14m$$

Multiply both sides by d, the ds on the RHS cancel to leave:

Divide both sides by E, the Es on the LHS cancel to leave:

Put the numbers in:

# Calculating SWR

$$SWR = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_{forward} + V_{reverse}}{V_{forward} - V_{reverse}}$$

Q60. An RF signal of 100V is travelling in the forward direction, 15% is reflected back to the transmitter. What is the resulting VSWR

Answer:

$$SWR = \frac{V_{forward} + V_{reverse}}{V_{forward} - V_{reverse}}$$
$$SWR = \frac{100 + 15}{100 - 15}$$
$$SWR = \frac{115}{85}$$
$$SWR = 1.35:1$$

Q61. An antenna results in half the incident power being reflected back to the transmitter, what will the SWR be?

$$SWR = \frac{V_{forward} + V_{reverse}}{V_{forward} - V_{reverse}}$$

But, here we are dealing with 'power' so we need to develop a formula for that.

Remember that  $P = \frac{V^2}{R}$  and since R will remain the same, multiply both sides of the equation by  $\sqrt{}$  and we are left with  $\sqrt{P}=V$ 

We can now rearrange the VSWR formula to a PSWR formula:

$$SWR = \frac{\sqrt{P_{forward}} + \sqrt{P_{reverse}}}{\sqrt{P_{forward}} - \sqrt{P_{reverse}}}$$

Or you can simplify this to:

$$SWR = \frac{1 + \sqrt{\frac{P_{reverse}}{P_{forward}}}}{1 - \sqrt{\frac{P_{reverse}}{P_{forward}}}}$$
$$SWR = \frac{1 + \sqrt{\frac{0.5}{1}}}{1 - \sqrt{\frac{0.5}{1}}}$$
$$SWR = \frac{1 + 0.707}{1 - 0.707}$$
$$SWR = \frac{1.707}{0.293}$$
$$SWR = 5.83:1$$

Q62. If a transmission line carries 81 Watts in the forward direction and 9 Watts in the reverse direction, what is the SWR on the line?

$$SWR = \frac{\sqrt{P_{forward}} + \sqrt{P_{reverse}}}{\sqrt{P_{forward}} - \sqrt{P_{reverse}}}$$
$$SWR = \frac{\sqrt{81} + \sqrt{9}}{\sqrt{81} - \sqrt{9}}$$
$$SWR = \frac{9 + 3}{9 - 3}$$
$$SWR = \frac{12}{6} = 2:1$$

# **Calculating Return Loss**

- $Return Loss = 10Log_{10} \frac{Reflected power}{Incident power}$ (dB)
- Q63. If a transmitter produces 100W and 5W is reflected at the antenna, what is the return loss?

Return Loss =  $10Log_{10}$  <u>Reflected power</u> Incident power

Return Loss =  $10 \text{Log}_{10} \underline{5}$ 100

Return Loss =  $10Log_{10} 0.05$ 

Return Loss = 10x1.3 = -13dB (note the minus sign)

Q64. If an antenna exhibits a return loss of 16dB, what will be the loss measured at the transceiver if the line loss is 2.5dB?

(Transceiver) Return Loss = Antenna return loss + (2 x Line loss) = 16dB + (2x2.5)= 21dB

Q65. An antenna exhibits a return loss of 22dB when measured in the shack, if the feeder loss is 3.5 dB what is the return loss at the antenna connection?

(Transceiver) Return Loss = Antenna return loss + (2 x Line loss)

But here, the scenario is reversed since we are looking for antenna return loss so:

Antenna return loss = (Transceiver) Return Loss – (2 x Line loss) = 22dB – (2 x 3.5dB) = 15dB

# Calculating Power Gain/Loss

$$Gain(dB) = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) \qquad \qquad Gain(dB) = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

Q66. An amplifier produces 60µV from an input of 15µV, what is its power gain?

$$Gain = 20 \log_{10} \left( \frac{V_{out}}{V_{in}} \right)$$
$$Gain = 20 \log_{10} \left( \frac{60}{15} \right)$$
$$Gain = 20 \log_{10} 4$$
$$Gain = 20 \times 0.6$$
$$Gain = 12 dB$$

Q67. If an amplifier has a gain of 12dB, what input voltage would be required to produce an output of 60µV?

$$Gain = 20 \log_{10} \left( \frac{V_{out}}{V_{in}} \right)$$
$$\left( \frac{V_{out}}{V_{in}} \right) = A \log_{10} \left( \frac{Gain(dB)}{20} \right)$$
$$\frac{60\mu V}{V_{in}} = A \log_{10} \left( \frac{12}{20} \right)$$
$$\frac{60\mu V}{V_{in}} = A \log_{10} \left( 0.6 \right)$$
$$\frac{60\mu V}{V_{in}} = 4$$
$$\frac{60\mu V}{4} = V_{in} = 15\mu V$$

Q68. An amplifier produces 50 Watts from an input of 3 Watts, what is its power gain?

$$Gain(dB) = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$$
$$Gain(dB) = 10 \log_{10} \left( \frac{50}{3} \right)$$
$$Gain(dB) = 10 \times 1.22$$
$$Gain(dB) = 12.2dB$$

The gain of a Yagi antenna is often quoted as "the increase in signal (in the forward direction) compared to that of a dipole". This expressed using dBd where the last d indicates dipole.

Or

 $Gain=10log \left( \frac{Power from Yagi}{Power from dipole} \right) \quad (dBd)$ 

Q69. If the radiated power from a Yagi antenna is 8 times greater than that of a dipole, what is it's gain in dBd?

 $Gain = 10 \times \log 8$  $Gain = 10 \times 0.9$ Gain = 9dBd

# The dBWatt (dBW)

You will notice that your licence indicates maximum power limits using dBW, this is a good example to work through for practice in converting to the more familiar power levels.

1W	0dBW
32W	15dBW
40W	16dBW
100W	20dbW
160W	22dBW
400W	26dBW

The output powers are all with reference to 1W so for instance, if you increase 1W by 15dB you have a figure of 32W.

Determining this using the 3 & 10 rule may give us some idea:

 $1W \ge 10dB = 10W$ ,  $\ge 3dB = 20W$ ,  $\ge 3dB = 40W$ (So a total of 16dBW = 40W)

1W x 10dB = 10W, x 10dB = 100W (So a total of 20dBW = 100W)

1W x 10dB = 10W, x 10dB = 100W, x 3dB =200W (So a total of 23dBW = 200W)

1W x 10dB = 10W, x 10dB = 100W, x 3dB =200W, x 3dB = 400W (So a total of 26dBW = 400W)

15dBW may make you think a little, as it is not obviously a part of the 3 & 10 rule but if you consider that every time you increase by 3dB you double the power, then 15dB is simply 5 times 3dB, giving 1W x2 x2 x2 x2 x2, which is equal to x32, or in this case 32W.

Q70. A transmitter has an output of 18dBW, what is the RF power?

Q71. A transmitter has an output of 29dBW what is the RF power?

 $1W \ge 10W, \ge 10W, \ge 100W, \ge 30W, \ge 200, \ge 30W = 200, \ge 30W$ (So a total of 29dB gives 800W)

Q72. A transmitter has an output of 4dbW what is the RF power?

10dBW - 6dB = 4dBW

+6dB is 3dB + 3dB which is equivalent to  $2 \ge 2 = x4$ , so minus 6db equates to  $\frac{1}{4}$ 

So <sup>1</sup>/<sub>4</sub> of 10dBW (10W) =2.5W

# **Calculating transistor gain \beta** $\beta = \frac{I_{\rm C}}{I_{\rm B}}$

Q73. If a transistor has  $\beta$  of 150 and the base current is 80µA, what is the collector current?

$$I_{c} = \beta \times I_{B}$$
$$I_{c} = 150 \times 80 \times 10^{-6}$$
$$I_{c} = 12 \times 10^{-3}$$
$$I_{c} = 12mA$$

Q74. If a transistor has collector current of 22ma and base current of 65 $\mu$ A, what is the  $\beta$ ?

$$\beta = \frac{I_C}{I_B}$$
$$\beta = \frac{22 \times 10^{-3}}{65 \times 10^{-6}}$$
$$\beta = 338$$

# Calculating Q Factor & Bandwidth

Q75. A medium wave receiver is tuned to 606 kHz and is receiving an AM signal of bandwidth 6 kHz, what would be a suitable Q factor for the tuned circuit?

$$Q = \frac{f_c}{f_u - F_L}$$

$$Q = \frac{centre.frequency}{bandwidth}$$

$$Q = \frac{606 \times 10^3}{6 \times 10^3}$$

$$Q = 101$$

Q76. In an FM transmission system the maximum audio frequency is 3 kHz, what is the bandwidth of a 5 kHz deviation signal?

 $BW = 2(AF_{max} + \Delta f)$ BW = 2(3kHz + 5kHz)BW = 16kHz

# Feeders and Antennas

Q77. A 75 $\Omega$  patch lead  $\lambda/4$  long, is used for connection to an oscilloscope, the lead is terminated with a load of 50 $\Omega$ , what will the impedance be at the free end?

$$Z_O^2 = Z_{in} \times Z_{out}$$
$$Z_{in} = \frac{Z_O^2}{Z_{out}}$$
$$Z_{in} = \frac{75 \times 75}{50}$$
$$Z_{in} = 112.5\Omega$$

Q78. A half wavelength of  $50\Omega$  feeder is terminated by a load of  $82\Omega$ , what will be the input impedance of the feeder?

Every half wavelength will repeat the load impedance so the input impedance will be  $82\Omega$ 

Q79. To minimize interference from a 2m transmitter into a broadcast FM receiver, a  $\lambda/4$  coaxial stub cut (from RG58U) for 145MHz is connected across the receiver coax down-lead, what is the physical length of the stub?

$$\lambda = \frac{300}{f(MHz)}$$
  

$$\lambda = \frac{300}{145}$$
  

$$\lambda = 2.069m$$
  

$$\frac{\lambda}{4} = \frac{2.069}{4} = 0.517m$$
  

$$0.517 \times 0.67(VelocityFactor) = 0.346m(34.6cm)$$

# Phase locked loop

Q80. If the reference crystal for a phase locked loop is 6MHz, the fixed divider is set to 6000 and the VCO set to divide by 10108, what will the output frequency be?

 $\begin{aligned} f_{out} &= f_{crystal} \times \frac{N}{A} & \dots & \text{Where N=VCO divider} \\ f_{out} &= 6 \times 10^6 \times \frac{10108}{6000} \\ f_{out} &= 10.108 \times 10^6 \\ f_{out} &= 10.108 MHz \end{aligned}$ 

Q81. What is the frequency step of the crystal in Q80, if the fixed divider is set to 60000?

$$f_{step} = \frac{f_{crystal}}{A}$$
$$f_{step} = \frac{6 \times 10^6}{60000}$$
$$f_{step} = 100 Hz$$

# Measurements

Oscilloscopes are widely used in RF measurements and are designed for frequencies up to 20-30MHz within the amateur market.

The oscilloscope consists of a cathode ray tube (CRT) with a grid of vertical and horizontal lines in front of it (called the graticule). Inside the CRT there is a vacuum. The electron beam emitted by the heated cathode at the rear of the tube is accelerated and focussed by one or more anodes and hits the front of the tube and produces a bright spot on the phosphorescent screen. The beam is deflected by voltages, applied to two sets of plates within the tube. The horizontal deflection plates (X-plates) produce 'side to side' movement. These X-plates are linked to the time

base that produces a saw tooth waveform. During the rising part of the sawtooth, the spot is driven at a uniform rate across the screen from left to right. On the falling part of the sawtooth, the beam returns rapidly from right to left. During this return, the spot is blanked out so nothing appears on the screen.

The time base generates the X-axis of the V/t graph

The slope of the rising part of the sawtooth varies with frequency and can be adjusted using the timebase control, marked TIME/DIV, this changes the scale of the horizontal X axis.

The fact that the screen is divided into squares, often 1cm, allows the horizontal scale to be expressed in seconds, milliseconds or microseconds per division or per cm, (s/Div, ms/DIV &  $\mu$ s/DIV).

The Y amplifier is linked to the Y plates, giving the Y or vertical axis of the V/t graph. The overall gain of the amplifier is set using the VOLTS/DIV control in order to adjust the display within the graticule for ease of interpretation. The vertical scale is usually in V/DIV or mV/DIV. The trigger circuit is used to delay the time-base waveform, so that the same section of the waveform is displayed on the screen each time the spot moves across; the effect of this is to produce a stable waveform on the screen which is easy to read.



Fig.15 The oscilloscope using a cathode ray tube

Q82. When using an oscilloscope, the vertical amplifier is set to 50mV per division, what is the peak to peak voltage of the signal waveform below?



Q83. If the oscilloscope timebase is set to  $20\mu$ S/cm, what is the frequency of the waveform?



Frequency (f) =1/period (t) and on inspection we can see that one cycle occupies 4 divisions on the graticule.

So 
$$t = 4 \times 20 \mu S$$
  
 $t = 80 \mu S$ 

And putting this into the frequency equation gives:

Q84. The oscilloscope graticule has 10 1cm divisions, in order to display three cycles of a 20kHz signal, what should the timebase be set to?



$$period(t) = \frac{1}{20 \times 10^3} = 50 \times 10^{-6} = 50 \mu S$$

We wish to display 3 periods so we need  $3x50=150\mu$ S (total).

The graticule has 10 1cm divisions so  $150/10=15\mu$ S means we need to use the  $20\mu$ S/cm Time/Div setting to capture the three cycles

Q85. A 270k $\Omega$  resistor is connected in series with a 50µA meter movement, what will the meter read at full scale deflection?

$R \ge 270k\Omega$	$V = I \times R$ $V = 50 \times 10^{-6} \times 270 \times 10^{3}$ $V = 13.5V$	Key sequence is: 50 X10 <sup>x</sup> (-) 6 X 270
Μ 🗩 50μA		270 X10 <sup>x</sup> 3 =13.5

# Transposing the Resonant frequency formula for L or C

 $f_r = \frac{1}{2\pi\sqrt{LC}}$  ..... (where  $f_r$  is the resonant frequency)

Assuming we need to make L the subject of the equation on the left hand side (LHS)

Multiply both sides by  $\sqrt{LC}$ 

$$\sqrt{LC} \times f_r = \frac{1 \times \sqrt{LC}}{2\pi \sqrt{LC}}$$

these cancel out, leaving:

$$\sqrt{LC} \times f_r = \frac{1}{2\pi}$$

Next, divide both sides of the equation by  $f_r$ 

$$\frac{\sqrt{LC}}{f_r} \times f_r = \frac{1}{2 \times \pi \times f_r}$$

and now the  $f_r$  components cancel out on the left side of the equation to leave:

$$\sqrt{LC} = \frac{1}{2 \times \pi \times f_r}$$

To get rid of the square root we need to square it, (the square and square root cancel). Don't forget, we need to square BOTH sides of the equation!

$$\left(\sqrt{LC}\right)^2 = \left(\frac{1}{2 \times \pi \times f_r}\right)^2$$

Then multiply out the contents of the brackets on the RHS. Since anything squared is simply multiplied by itself

$$LC = \frac{1 \times 1}{2 \times 2 \times \pi \times \pi \times f_r \times f_r}$$

This now leaves us with:

$$LC = \frac{1}{4 \times \pi^2 \times f_r^2}$$

Now we need to leave L on the left and move C into the RHS of the equation by dividing both sides by C

$$\frac{L\mathcal{C}}{\mathcal{C}} = \frac{1}{4 \times \pi^2 \times f_r^2 \times C}$$

Here, the Cs cancel out to leave:

$$L = \frac{1}{4 \times \pi^2 \times f_r^2 \times C}$$

If we ignore the '**X**' signs we are left with:

$$L = \frac{1}{4\pi^2 f_r^2 C}$$

You can make 'C' the subject of the equation by dividing both sides of the equation by L as the final step in the process.